15401

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15401

M. Tech. 1 - Sem. (Main) Exam., Dec. - 2018

Digital Communication

1MDC1 Signal Theory

Time: 3 Hours **Maximum Marks: 100**

Min. Passing Marks: 33

Instructions to Candidates:

*Attempt any **five questions,** Marks of questions are indicated against each question. Draw neat and comprehensive sketches wherever necessary to clearly illustrate your answer. Assume missing data suitably if any and specify the same. Use of following supporting material is permitted during examination. (Mentioned in form No. 205)*

1. NIL 2. NIL

Q.1 (a) A random process X (t) has an autocorrelation function [10]

$R_{xx}(\tau)=A^2+Be^{|\tau|}$

Where A and B are positive constants. Find the mean value of the response of a system having an impulse response

$h(t)=\begin{cases} te^{-Wt} & 0<t \\ 0 & \text{when } t<0 \end{cases}$ Where W is real positive constant, for which X (t) is input.

(b) Consider the random process X (t) = Y cos ωt t > 0 where ω is a constant and Y is a uniform r. v. over (0,1). [10]

(i) Find E [X (t)],

(ii) Find the autocorrelation function $R_x(t,s)$ of X (t) and Find the auto covariance function $K_x(t, s)$ of X (t).

Q.2 (a) Two random processes are given by [10]

$X(t) = A\cos(\omega t + \theta)$, $Y(t) = A\sin(\omega t + \theta)$ where A and ω are constants and θ is a uniform random variable over $(0, 2\pi)$. Find the cross correlation function of X (t) and Y (t) and verify $R_{xy}(-\tau) = R_{yx}(\tau)$.

(b) Let Y (t) be the output of an LTI system with impulse response h (t) when a WSS random process X (t) is applied as input. Show that: [10]

(i) $S_{XY}(\omega) = H(\omega)\, SX(\omega)$

(ii) $SY(\omega) = H^*(\omega)\, S_{XY}(\omega)$.

Q.3 (a) A random process has the power spectrum, find the average power in the process. [10]

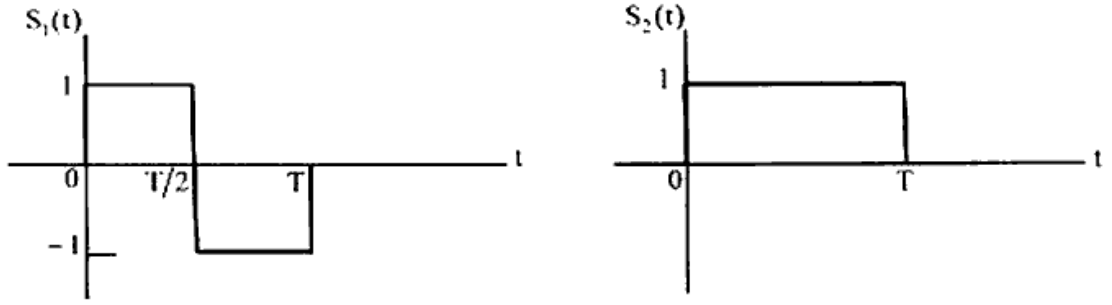
$$S_{xx}(\omega) = \frac{6\omega^2}{(1+\omega^2)^3}$$

(b) A WSS random process X (t) is applied to the input of an LTI system whose impulse response is $3te^{-2t}$. Find the mean value of the output Y(t) of the system, if $E\{X(t)\} = 2$. [10]

Q.4 (a) What is Optimum filtering? Derive an expression for transfer function of matched filter for colored noise. [12]

(b) White noise for which $N_0/2 = 10^{-8} / 2\pi$ and a signal $X(t) = Wte^{-t}$ for $0<t<2/W$; and 0; otherwise, is applied to a matched filter. What ratio of output peak signal power to average noise power can be achieved if $W = 5\,(10^6)$ rad/s? [8]

Q.5 (a) Figure shows a orthogonal sets of signals. Construct a signal constellation for $S_1(t)$ and $S_2(t)$. [10]



(b) If X (t) is a WSS random process and has a m. s. derivative X′ (t), then show that [10]

(i) $R_{xx}(\tau)= \frac{d}{d\tau} R_x(\tau)$

(ii) $R_x(\tau)= - \frac{d^2}{d\tau^2} R_x(\tau)$

Q.6 Briefly explain the following Random processes with suitable examples [4×5=20]

(i) Gaussian Process

(ii) Ergodic Process

(iii) Wiener Process

(iv) Poisson's Process

Q.7 Write the short notes on any two - [2×10=10]

(i) K-L Expansion

(ii) Gram – Schmidt Orthogonalization

(iii) Shot Noise Process

Q.8 (a) Discuss the Markov Processes with suitable examples. [10]

(b) Consider a Markov chain with state space {0, 1} and transition probability matrix [10]

$$P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

(i) Show that state 0 is recurrent

(ii) Show that state 1 is transient
