410701/410801

Roll No. ____________

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B. Tech. IV - Sem. (Main / Back) Exam., (Academic Session 2021- 2022)

Electronics & Communication Engineering

4EC2 – 01/4EI2 – 01 Advanced Engineering Mathematics - II

Common to ECE & EIC

Time: 2½ Hours **Maximum Marks: 120**

Min. Passing Marks:

Instructions to Candidates:

Part – A: *Short answer questions (up to 25 words) 6 × 3 marks = 18 marks. Candidates have to answer six questions out of ten.*

Part – B: *Analytical/Problem solving questions 3 × 10 marks = 30 marks. Candidates have to answer three questions out of seven.*

Part – C: *Descriptive/Analytical/Problem Solving questions 3 ×24 marks = 72 marks. Candidates have to answer three questions out of five.*

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

PART – A

Q.1 Define invariant points of bilinear transformation.

Q.2 Define harmonic function, check whether $3x^2y - y^3$ is harmonic or not?

Q.3 State Cauchy's theorem.

Q.4 Write Legendre's differential equation.

Q.5 Give an example of, when a transformation is said to be conformal.

Q.6 Write generating function for Bessel's function.

Q.7 Write the matrix of the quadratic form $x^2 + 2y^2 + 3z^2 + 4xy + 5yz + 6zx$.

Q.8 Define characteristic polynomial with an example.

Q.9 Give an example of, when a set of vectors is said to be linearly independent.

Q.10 Define pole and residue at pole.

PART – B

Q.1 (a) Prove that a bilinear transformation maps circles into circles.

(b) Prove that a bilinear transformation preserves the cross ratio of four points.

Q.2 (a) If f (z) is an analytic function of constant modulus, show that f (z) is constant.

(b) If the potential function is $\log (x^2 + y^2)$, find the flux function and complex potential function.

Q.3 Prove that -

(a) $J_{\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi z}} \sin z$

(b) $J_{\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi z}} \cos z$

Q.4 By integrating around a unit circle, evaluate $\int_0^{2\pi} \frac{\cos 2\theta d\theta}{1-2a\cos\theta+a^2}$, where $a^2 < 1$.

Q.5 Prove that all the roots of $P_n(x) = 0$ are real and lying between -1 and 1.

Q.6 Find the Eigen values and Eigen vectors of the matrix $A=\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$.

Q.7 Extend $\{(2, 3, -1),(1, -2, -4)\}$ to an orthogonal basis of the Euclidean space R^3 with standard inner product and then find the associated orthonormal basis.

PART – C

Q.1 (a) Show that $f(z) = \sinh z$ is an analytic function. Find its derivative.

(b) If $f(z)$ is a regular function of z, prove that -

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

Q.2 Prove that -

(i) $J_{-n}(z) = (-1)^n J_n(z)$, when n is a positive integer.

(ii) $J_n(-z) = (-1)^n J_n(z)$, for positive or negative integer n.

(iii) $J_n(z)$ is an even or odd function according as n is even or odd respectively.

Q.3 State and prove the orthogonality property of Legendre's polynomial.

Q.4 (a) Reduce the quadratic form $5x^2 + y^2 + 10z^2 - 4yz - 10zx$ to the canonical form.

(b) Find the QR-decomposition of the matrix $A = \begin{bmatrix} -2 & 3 \\ 5 & 7 \\ 2 & -2 \\ 4 & 6 \end{bmatrix}$

Q.5 Find $\int_C z^2 dz$, where C is the curve passing through the points 1+i and 2(1+2i) and specified as -

(i) the arc $y = x^2$

(ii) the straight line joining the points 1+i and 2(1+2i)

(iii) the arc of $z = t + it^2$
