4E030121

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B. Tech. IV Semester End-Term Examination (Main), June-2022

Branch: Electronics & Communication Engineering

4EC2-01: Random Variable & Stochastic Process

Time: 3 Hours **Maximum Marks: 140**

Instructions to Candidates:

The question paper is divided in three parts A, B & C.

(i) **Part-A:** 7 Basics/Fundamentals related questions (without choice).

(ii) **Part-B:** 5 Numerical/Analytical questions (with internal choice i.e. attempt one question either A or B from each question).

(iii) **Part-C:** 5 Descriptive/Analytical/Problem Solving/Design questions **(attempt any 3 out of 5).**

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination:

1. Nil 2. Nil

PART-A

(Basics/Fundamentals related questions)

All questions are compulsory

Q.1 (a) Define probability with suitable example. [4]

(b) Explain the power set. [4]

(c) What do you mean by conditional probability? [4]

(d) Check whether the following function serve as probability mass function [4]

$$P(X=x) = \frac{x-2}{2}, \forall x = 1,2,3,4.$$

(e) The joint probability mass function of (x, y) is given by [4]
$p(x, y) = k(2x + 3y),\ x = 0, 1, 2\ ; y = 1, 2, 3$ then find:
(i) k ; (ii) Marginal probability distribution of X.

(f) Define stochastic process. [4]

(g) Define ergodicity. [4]

PART-B

(Numerical/Analytical questions)

Q.2 (A) A bag contains 4 bad and 6 good mobile phones. Two are drawn out from the bag at a time. One of them is tested and found to be good. What is the probability that the other phone is also good? [8]

OR

(B) State and prove Baye's theorem. [8]

Q.3 (A) Calculate the mean and variance of binomial distribution. [8]

OR

(B) Prove that the Poisson distribution is the limiting case of Binomial distribution. [8]

Q.4 (A) If X and Y are two independent exponential random variables with parameter 1. Let U=X+Y and V=X-Y. Then find the joint and marginal probability density function of U and V. [8]

OR

(B) Verify the property: [8]
$P(x_1 < X \leq x_2, Y \leq y) = F_{XY}(x_2, y) - F_{XY}(x_1, y)$

Q.5 (A) If the WSS process [X(t)] is given by $X(t) = 10\cos(100t + \theta)$ where θ is uniformly distributed over $(-\pi, \pi)$. Prove that [X(t)] is correlation ergodic. [8]

OR

(B) Consider a continuous random variable X, prove that: [8]
$E(X) = \int_0^{\infty}[1 - F_x(X)]dX - \int_{\infty}^{0} F_X(x)dx$, where E(X) denotes the expectation of random variable X.

Q.6 (A) The power spectrum of noise N(t) is defined as: [8]

$$S_{NN}(\omega) = \begin{cases} \frac{N_0}{2} ; -w \leq \omega \leq w \\ 0 ; \text{elsewhere} \end{cases}$$

Determine the autocorrelation function of N(t) and draw the plot of $R_{NN}(\tau)$ versus τ.

OR

(B) Let {X(t)} and {Y(t)} are two random processes given by: [8]
$X(t) = U\cos(w_0 + \theta)$ and $Y(t) = U\sin(w_0 + \theta)$ where U and w_0 are constants and θ is a random variable over $(0, 2\pi)$

PART-C

(Descriptive/Analytical/Problem Solving/Design questions)
(attempt any 3 out of 5) (Q.7 to Q.11)

Q.7 If $P(\frac{A}{B})$ is the conditional probability of A and B, then show that: [24]

(i) $P(\frac{A}{B}) \geq 0$

(ii) $P\left(\frac{S}{B}\right) = 1$

(iii) $P\left(\frac{A_1 \cup A_2}{B}\right) = P\left(\frac{A_1}{B}\right) + P\left(\frac{A_2}{B}\right)$ if $A_1 \cap A_2 = \emptyset$

(iv) If $P\left(\frac{A}{B}\right) > P(A)$ then prove that: $P\left(\frac{B}{A}\right) > P(B)$.

Q.8 Fine the mean, variance and moment generating function of normal distribution. [24]

Q.9 If (x,y) is a two dimensional random variable uniformly distributed i.e., $f(x, y) = K$ over the triangular region R bounded by $y = 0$, $x = 3$ and $y = \frac{4x}{3}$ then find:
(i) E (X) (ii) Var (X) (iii) E (Y) (iv) Var (Y). [24]

Q.10 State and explain the central limit theorem in detail with appropriate mathematical equations. [24]

Q.11 Let Y(t) be the output of an LTI system with impulse response h(t) when a WSS random process X(t) is applied as input, show that: [24]

(i) $S_{xy}(w) = H(w)S_x(w)$

(ii) $S_y(w) = H^*(w)S_{xy}(w)$

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