11501

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11501
B. Tech. I Sem. (Back) Exam., May - 2019
BSC
1FY2 – 01 Engineering Mathematics - I

Time: 3 Hours **Maximum Marks: 160**

Instructions to Candidates:

Part – A: *Short answer questions (up to 25 words) 10 × 3 marks = 30 marks. All ten questions are compulsory.*

Part – B: *Analytical/Problem solving questions 5 × 10 marks = 50 marks. Candidates have to answer five questions out of seven.*

Part – C: *Descriptive/Analytical/Problem Solving questions 4 × 20 marks = 80 marks. Candidates have to answer four questions out of five.*

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

PART - A

Q.1 What is the value of $\Gamma\left(-\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)$?

Q.2 Write the formula of surface area of solid of revolution when the revolution is about x – axis.

Q.3 Find whether series $\Sigma \frac{n}{n+10}$ is convergent or not?

Q.4 Give an example of a divergent series whose sum is convergent.

Q.5 State Parseval's theorem.

Q.6 Find sum of Fourier series of periodic function f(x) at x = 2 , where

$$f(x) = \begin{cases} 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$$

Q.7 Give an example of function which is not continuous at origin although partial derivatives exist.

Q.8 Evaluate $\int_0^{\pi}\int_0^{x} \sin y\,dy\,dx$.

Q.9 Write the formula of centre of gravity of plane lamina occupying an area A in the xy – plane and having density p = f (x, y).

Q.10 State the Gauss divergence theorem.

PART –B

Q.1 Use gamma function, show that –

$$\int_0^{\infty} \frac{1}{1+y^4}\,dy = \frac{\pi}{2\sqrt{2}}$$

Q.2 Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1^2.5^2.9^2...(4n-3)^2}{4^2.8^2.12^2...(4n)^2}$$

Q.3 Find Fourier series of x^2 in $(-\pi, \pi)$, and use Parseval's identity to prove

$$\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$$

Q.4 Find the equation of the tangent plane and normal to the surface xyz = 4 at the point (1, 2, 3).

Q.5 Find the minimum value of the function $f(x) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$

Q.6 Change the order of integration and hence solve

$$\int_0^1 \int_{e^x}^{e} \frac{1}{\log y} dxdy$$

Q.7 Verify Stokes' theorem when $\vec{F} = x^2 i + xy\, j$, where C is the perimeter of the square in xy – plane whose sides are along the lines x = 0, y =0, x = a and y = a.

PART –C

Q.1 (a) Find the volume generated by revolving the portion of the parabola $y^2 = 4ax$ cut off by its latus rectum about y – axis.

(b) Expand sin x in powers of (x - π/2) by Taylor's series.

Q.2 (a) Prove that –

$$e^x = 1 + \tan x + \frac{1}{2!}\tan^2 x - \frac{1}{3!}\tan^3 x - \frac{7}{4!}\tan^4 x + \cdots$$

(b) Find the Fourier series to represent $f(x) = x - x^2$ in $-1 < x < 1$.

Q.3 (a) Use Lagrange's method of multiplier to divide 24 into three parts such that the continued product of the first, square of the second and cube of the third may be maximum.

(b) Find the half range sine and cosine series of f(x) = x in (0, π).

Q.4 (a) If $\vec{r} = xi + yj + zk$, then find:

(i) div $(r^n \vec{r})$

(ii) curl $(r^n \vec{r})$

(b) Find the area of the region lying in first quadrant enclosed by the circle $x^2 + y^2 = a^2$ and the line $x + y = a$ by double integration.

Q.5 (a) Apply Green's theorem to evaluate $\int_C (x^2 = \cosh y)dx + (y + \sin x)\,dy$, where C is the rectangle with vertices (0, 0), (π, 0), (π, 1), (0, 1).

(b) If $u = e^{xyz}$, then prove that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$
