11501

Roll No. __________

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B. Tech. I - Sem. (Main/Back) Exam., Dec. - 2019

BSC

1FY2-01 Engineering Mathematics - I

Time: 3 Hours **Maximum Marks: 160**

Instructions to Candidates:

Part – A: *Short answer questions (up to 25 words) 10 × 3 marks = 30 marks. All ten questions are compulsory.*

Part – B: *Analytical/Problem Solving questions 5 × 10 marks = 50 marks. Candidates have to answer five questions out of seven.*

Part – C: *Descriptive/Analytical/Problem Solving questions 4 × 20 marks = 80 marks. Candidates have to answer four questions out of five.*

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitable be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting materials is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

PART - A

Q.1 Evaluate: $\dfrac{\sqrt{\frac{1}{3}}\sqrt{\frac{5}{6}}}{\sqrt{\frac{2}{3}}}$ [3]

Q.2 Evaluate: $\int_0^2 \frac{x^2}{\sqrt{(2-x)}}dx$ [3]

Q.3 The Part of the Parabola $y^2 = 4ax$ cut off by the latus rectum revolves about the tangent at the vertex. Find the Volume of the reel thus generated. [3]

Q.4 Find the Surface of the solid generated by the revolution of the asteroid $x = a\cos^3 t$, $y = a\sin^3 t$ about the x-axis. [3]

Q.5 Show that the following function is discontinuous at (0, 0): [3]

$$f(x,y) = \begin{cases} \frac{xy^3}{x^2+y^6}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Q.6 If $u = \sec^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, then prove that: [3]

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$$

Q.7 Find the derivative of the function at P_0 in the direction of u: [3]

$f(x, y) = 2xy - 3y^2$, P_0 (5,5), $u = 4\hat{i} + 3\hat{j}$

Q.8 Find the tangent plane to the surface $z = x\cos y - ye^x$ at (0, 0, 0). [3]

Q.9 Find the max or min value of the following function: [3]

$$u = xy + \frac{a^3}{x} + \frac{a^3}{y}$$

Q.10 Evaluate: $\iiint xyz \, dx\, dy\, dz$ [3]

where the region of integration is the complete ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$$

PART – B

Q.1 Show that a Sequence is convergent iff (⇔) it is a Cauchy Sequence. [10]

Q.2 Test the convergence of the following series: [10]

$$\frac{2x}{1^2} + \frac{3^2x^2}{2^3} + \frac{4^3x^3}{3^4} + \frac{5^4x^4}{4^5} + \ldots\ldots\ldots\ldots$$

Q.3 Find half –range sine series for the function $f(x) = x$ in the interval $0 < x < 2$. [10]

Q.4 If $r = |\vec{r}|$, where $\vec{r} = x\hat{i} + y\hat{j} + 3\hat{k}$, then prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$. [10]

Q.5 Find the constant a so that $\vec{V}$ is a conservative vector field, where [10]

$\vec{V} = (axy - z^3)\hat{i} + (a-2)x^2\hat{j} + (1-a)xz^2\hat{k}$. Calculate its potential and work done, in moving a particle from (1, 2, -3) to (1, -4, 2) in the field.

Q.6 Evaluate the following double integral by changing the order of integration: [10]

$$\int_0^{\frac{a}{\sqrt{2}}} \int_x^{\sqrt{a^2-x^2}} y^2 \, dx \, dy$$

Q.7 Find the center of gravity (C.G.) of the area in the positive quadrant of the curve: [10]

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

PART - C

Q.1 Show that: [20]

$$\int_0^{\frac{\pi}{2}} \sin^m\theta \cos^n\theta \, d\theta = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\,\Gamma\left((m+n+2)/2\right)}$$

Q.2 Find the Fourier series of the function $f(x) = x + x^2$ in the interval $(-\pi, \pi)$ and show that

$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots$ [20]

Also find the sum of the series when $x = \pm\pi$.

Q.3 Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial n^2}$ [20]

where $x = \xi\cos\alpha - n\sin\alpha$, $y = \xi\sin\alpha + n\cos\alpha$

Q.4 Find the volume bounded above by the sphere $x^2 + y^2 + z^2 = 2a^2$ and the paraboloid

$az = x^2 + y^2$. [20]

Q.5 Verify Gauss's divergence theorem given that $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is surface of the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1. [20]

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