

11501

Roll No. \_\_\_\_\_

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B. Tech. I - Sem. (Main) Exam., Dec. - 2018

BSC

1FY2 – 01 Engineering Mathematics - I

Time: 3 Hours

Maximum Marks: 160

*Instructions to Candidates:*

*Part – A: Short answer questions (up to 25 words) 10 × 3 marks = 30 marks. All ten questions are compulsory.*

*Part – B: Analytical/Problem solving questions 5 × 10 marks = 50 marks. Candidates have to answer five questions out of seven.*

*Part – C: Descriptive/Analytical/Problem Solving questions 4 × 20 marks = 80 marks. Candidates have to answer four questions out of five.*

*Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.*

*Use of following supporting material is permitted during examination. (Mentioned in form No. 205)*

1. NIL

2. NIL

**PART - A**

Q.1 Evaluate the integral by using the Gamma function: [3]

$$\int_0^{\infty} e^{-x^2} dx$$

Q.2 Examine the convergence of the sequence  $\{a_n\}$ , where  $a_n = \sqrt{n} (\sqrt{n+1} - \sqrt{n})$  [3]

Q.3 State Parseval's theorem. [3]

Q.4 Show that  $(x, y) \rightarrow (0, 0) \left[ \frac{x}{\sqrt{x^2+y^2}} \right]$  does not exist. [3]

Q.5 Find the equation of the tangent plane to [3]

$$z = x\sqrt{x^2+y^2} + y^3 \text{ at } (-4, 3).$$

Q.6 Find the values of a, b, c, so that  $\vec{A}$  is irrotational, where [3]

$$\vec{A} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$

Q.7 Evaluate  $\int_C \vec{F} \cdot d\vec{r}$ , where  $\vec{F} = x^2y^2\hat{i} + y\hat{j}$  and C is the curve  $y^2 = 4x$  in the xy - plane from (0, 0) to (4, 4). [3]

Q.8 Evaluate the double integration: [3]

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2} dx dy$$

Q.9 Prove by using Logarithmic series: [3]

$$\frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2^2} + \frac{1}{3} \cdot \frac{1}{2^3} - \frac{1}{4} \cdot \frac{1}{2^4} + \dots = \log \frac{3}{2}$$

Q.10 State Green's theorem in the plane. [3]

**PART - B**

Q.1 Evaluate  $\int_0^{\infty} \frac{x^2(1+x^2)}{(1+x)^{10}} dx$  [10]

Q.2 Find the half - range cosine series for the function:  $f(x) = 2x - 1$  for  $0 < x < 1$ . [10]

Q.3 Test the convergence of the series [10]

$$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots$$

For all positive values of  $x$ .

Q.4 Show that the function: [10]

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & , \quad (x, y) \neq (0, 0) \\ 0 & , \quad (x, y) = (0, 0) \end{cases}$$

possess partial derivatives  $f_x(0, 0)$  and  $f_y(0, 0)$ , though it is not continuous at  $(0, 0)$ .

Q.5 Find the directional derivative of  $\phi(x, y, z) = x^2 - 2y^2 + 4z^2$  at  $(1, 1, -1)$  in the direction of the vector  $2\hat{i} + \hat{j} - \hat{k}$ . Also find the direction of the maximum directional derivative at  $(1, 1, -1)$ . [10]

Q.6 Evaluate  $\iiint_R (x + y + z) dx dy dz$ , where  $R : 0 \leq x \leq 1, 1 \leq y \leq 2, 2 \leq z \leq 3$ . [10]

Q.7 Evaluate the integral  $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dx dy$  by changing into polar co-ordinates. [10]

**PART - C**

Q.1 Find the surface and the volume of the spindle shaped solid formed by revolving the  
asteroid  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$  about x - axis. [20]

Q.2 Find the Fourier series for the function  $f(x) = x + x^2$  for  $-\pi < x < \pi$  [20]

Hence deduce  $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$

and  $\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

Q.3 Using Lagrange's multiplier method to obtain the extreme points of the function [20]

$$f(x, y, z) = x + y + z$$

Subject to  $x^2 + y^2 + z^2 = 1$

Find whether the extreme points are maxima or minima.

Q.4 Verify Gauss' divergence theorem, given that [20]

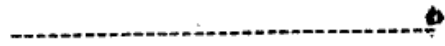
$$\vec{F} = 4xz \hat{i} - y^2 \hat{j} + yz \hat{k}$$

and S is the surface of the cube bounded by the planes

$$x = 0, x = 1, y = 0, y = 1, z = 0 \text{ and } z = 1$$

Q.5 Evaluate the integral by changing the order of integration [20]

$$\int_0^1 \int_0^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dx dy$$



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