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B. Tech. I - Sem. (Main) Exam., Dec. - 2018

BSC

1FY2 – 01 Engineering Mathematics - I

Time: 3 Hours **Maximum Marks: 160**

Instructions to Candidates:

Part – A: Short answer questions (up to 25 words) 10 × 3 marks = 30 marks. All ten questions are compulsory.

Part – B: Analytical/Problem solving questions 5 × 10 marks = 50 marks. Candidates have to answer five questions out of seven.

Part – C: Descriptive/Analytical/Problem Solving questions 4 × 20 marks = 80 marks. Candidates have to answer four questions out of five.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

PART - A

Q.1 Evaluate the integral by using the Gamma function: [3]

$$\int_0^\infty e^{-x^2}\,dx$$

Q.2 Examine the convergence of the sequence $\{a_n\}$, where $a_n = \sqrt{n}\left(\sqrt{n+1} - \sqrt{n}\right)$ [3]

Q.3 State Parseval's theorem. [3]

Q.4 Show that $\lim_{(x, y) \to (0,0)} \left[\frac{x}{\sqrt{x^2+y^2}}\right]$ does not exist. [3]

Q.5 Find the equation of the tangent plane to [3]

$z = x\sqrt{x^2+y^2} + y^3$ at (-4, 3).

Q.6 Find the values of a, b, c, so that $\vec{A}$ is irrotational, where [3]

$$\vec{A} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$

Q.7 Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = x^2y^2\hat{i} + y\hat{j}$ and C is the curve $y^2 = 4x$ in the xy – plane from (0, 0) to (4, 4). [3]

Q.8 Evaluate the double integration: [3]

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\, dx\, dy$$

Q.9 Prove by using Logarithmic series: [3]

$$\frac{1}{2} - \frac{1}{2}\cdot\frac{1}{2^2} + \frac{1}{3}\cdot\frac{1}{2^3} - \frac{1}{4}\cdot\frac{1}{2^4} + \ldots\ldots\ldots\ldots = \log\frac{3}{2}$$

Q.10 State Green's theorem in the plane. [3]

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PART – B

Q.1 Evaluate $\int_0^\infty \frac{x^2(1+x^2)}{(1+x)^{10}}\, dx$ [10]

Q.2 Find the half – range cosine series for the function: $f(x) = 2x - 1$ for $0 < x < 1$. [10]

Q.3 Test the convergence of the series [10]

$$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots$$

For all positive values of x.

Q.4 Show that the function: [10]

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

possess partial derivatives $f_x(0, 0)$ and $f_y(0, 0)$, though it is not continuous at (0, 0).

Q.5 Find the directional derivative of $\phi(x, y, z) = x^2 - 2y^2 + 4z^2$ at (1, 1, -1) in the direction of the vector $2\hat{i} + \hat{j} - \hat{k}$. Also find the direction of the maximum directional derivative at (1, 1, -1). [10]

Q.6 Evaluate $\iiint_R (x + y + z)\, dx\, dy\, dz$, where $R : 0 \le x \le 1, \quad 1 \le y \le 2, \quad 2 \le z \le 3$. [10]

Q.7 Evaluate the integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$ by changing into polar co-ordinates. [10]

PART – C

Q.1 Find the surface and the volume of the spindle shaped solid formed by revolving the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about x – axis. [20]

Q.2 Find the Fourier series for the function $f(x) = x + x^2$ for $-\pi < x < \pi$ [20]

Hence deduce $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

and $\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

Q.3 Using Lagrange's multiplier method to obtain the extreme points of the function [20]

$$f(x, y, z) = x + y + z$$

Subject to $x^2 + y^2 + z^2 = 1$

Find whether the extreme points are maxima or minima.

Q.4 Verify Gauss' divergence theorem, given that [20]

$$\vec{F} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$$

and S is the surface of the cube bounded by the planes

$x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$

Q.5 Evaluate the integral by changing the order of integration [20]

$$\int_0^1 \int_0^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dx\, dy$$
